Decomposition Based Evolutionary Algorithm with a Dual Set of Reference Vectors: Supplementary Material

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1) Regular Problems: Next we demonstrate the performance of DBEA-DS on 13 unconstrained benchmark problems from DTLZ [1] and WFG [2]: DTLZ1-DTLZ4 and WFG1-WFG9. The performances delivered by the proposed approach is compared in terms of HV in Table I and IGD in Table II with several recent and popular algorithms. The performances delivered by the other algorithms are obtained from [3].

Based on mean HV statistics listed in Table I, DBEA-DS performs at par with the best performing algorithm in 3 objective and 5 objective instances of DTLZ1, DTLZ2, DTLZ4, WFG2 and WFG4-WFG9 problems i.e. 10 out of 13 problems. The proposed approach delivers at par performance with the best performing algorithm in the context of 8 objective instances of WFG4-WFG9 i.e. 6 out of 13 problems. In the context of 8 objectives instances of DTLZ1 and DTLZ3, DBEA-DS performs significantly worse. It is important to note that, performance of different algorithms based on HV varies significantly based on different reference points as seen in [3]. Therefore, it is very difficult to judge the performance of an algorithm based on HV alone. Hence, IGD values are also presented below for these problems.

For IGD computation of these problems, reference sets have been obtained from the authors of [3] and results of the listed algorithms are also from the same source [3]. Based on the mean IGD statistics, DBEA-DS performs best in 3 objective instances of DTLZ1-DTLZ4, and 8 objective instances of WFG4-WFG8 problems. In the context of 5 and 8 objective instances of DTLZ1, DTLZ3 and WFG1, DBEA-DS performs significantly worse in terms of mean IGD values. However, the proposed approach delivers at par performance with the best performing algorithm in terms of mean IGD values in the context of other instances of other problems.

References

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TABLE I
MEAN HV STATISTICS FOR DTLZ AND WFG PROBLEMS

Problem	M	DBEA-DS	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1	3	**1.11929**	1.11508	1.11767	1.11913	1.11711	1.06842	0.39572	0.48149	1.07411
	5	1.40663	1.57677	1.57767	**1.57794**	1.57768	1.51186	0.50052	0.02284	0.00000
	8	0.00000	2.13770	**2.13788**	2.13730	2.13620	2.05463	0.96246	1.44289	0.00000
DTLZ2	3	0.73512	0.74336	0.74390	**0.74445**	0.74418	0.70168	0.33187	0.33100	0.69708
	5	1.15924	1.30317	1.30679	**1.30778**	1.30728	1.14598	0.61944	0.27191	0.67442
	8	1.17771	1.96916	1.97785	**1.97862**	1.97817	1.35469	0.68315	0.54410	0.00004
DTLZ3	3	0.73208	0.73300	0.73642	**0.73944**	0.73654	0.69553	0.33026	0.31397	0.69959
	5	0.42845	1.29894	1.30376	**1.30638**	1.30398	1.14475	0.60143	0.00750	0.00000
	8	0.00000	1.95007	1.96849	**1.97162**	1.74240	1.33166	0.66684	0.29765	0.00000
DTLZ4	3	0.73261	0.73221	0.71077	**0.74484**	0.48232	0.45889	0.17191	0.23377	0.70481
	5	1.20937	1.30839	**1.30878**	1.30876	1.20680	1.00426	0.42941	0.33457	1.00881
	8	1.17075	1.98025	1.98078	**1.98083**	1.86439	1.35100	0.71296	0.53303	0.00000
WFG1	3	0.54645	0.65088	0.70151	0.69393	0.67291	**0.92204**	0.73804	0.81622	0.75944
	5	0.52483	0.85608	1.14844	1.23809	1.34797	**1.51824**	1.36724	1.36241	1.03120
	8	0.54255	1.36206	1.88297	1.91925	1.73875	**2.05117**	1.85604	1.75472	1.51083
WFG2	3	1.21957	1.22359	**1.22945**	1.22193	1.11888	1.12990	1.12266	1.16687	1.20760
	5	1.58718	**1.59770**	1.59708	1.55672	1.52205	1.58417	1.42821	1.42081	1.58790
	8	1.61809	**2.13629**	2.12442	2.04619	2.01678	2.13569	2.11651	2.11529	2.13214
WFG3	3	0.80063	0.81929	0.81556	0.77295	0.75364	0.80041	0.48971	0.74146	**0.82967**
	5	0.84437	1.01000	1.02782	0.95386	0.89357	0.88322	0.71619	0.93099	**1.06314**
	8	0.99572	1.21146	1.11348	1.15306	0.74674	1.27479	0.92248	1.41331	**1.41857**
WFG4	3	0.68476	0.72867	**0.72949**	0.72031	0.68710	0.66650	0.34131	0.63483	0.67605
	5	1.22600	1.28496	**1.28736**	1.26067	1.15695	1.01300	0.71180	1.04810	1.07969
	8	1.84477	1.96402	**1.96426**	1.83751	1.19841	1.33398	0.95883	1.45141	1.40330
WFG5	3	0.67751	0.68658	**0.68706**	0.67698	0.65668	0.61681	0.27764	0.58174	0.65059
	5	1.20908	1.22187	**1.22209**	1.18965	1.11627	0.93276	0.58164	0.96542	1.06695
	8	1.78109	1.84995	**1.85027**	1.71196	1.27483	1.18970	0.96591	1.33675	1.39529
WFG6	3	0.65644	0.68696	**0.68698**	0.67923	0.65655	0.62307	0.28542	0.58469	0.64111
	5	0.82066	1.21978	**1.22284**	1.19424	1.04043	0.93460	0.55026	0.97587	1.01175
	8	1.61609	**1.84625**	1.84330	1.69055	0.71742	1.17924	0.63171	1.21597	1.27938
WFG7	3	0.67346	0.72894	**0.73099**	0.72126	0.61145	0.66659	0.33309	0.62859	0.68591
	5	0.91549	1.29190	**1.29548**	1.25983	1.07723	1.01449	0.63899	1.04794	0.97811
	8	1.89306	1.97138	**1.97353**	1.82024	0.83439	1.30773	0.71170	1.45307	1.22911
WFG8	3	0.61773	0.66560	**0.66687**	0.65741	0.62986	0.61394	0.24450	0.26792	0.61230
	5	1.04518	1.18225	**1.18354**	1.15376	0.95660	0.60364	0.46673	0.82273	0.96648
	8	1.65747	1.75970	**1.76647**	1.70621	0.30471	1.20786	0.67808	1.24044	1.28486
WFG9	3	0.62664	0.67519	**0.67978**	0.67146	0.57864	0.62177	0.25170	0.51403	0.62199
	5	0.97311	1.21058	**1.22122**	1.15493	1.02426	0.78608	0.53143	0.94420	0.92841
	8	1.07759	1.80911	**1.83678**	1.60407	0.97800	1.23897	0.72454	1.18318	1.07824

TABLE II
MEAN IGD STATISTICS FOR DTLZ AND WFG PROBLEMS

Problem	M	DBEA-DS	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1	3	**0.03946**	0.04362	0.04170	0.04138	0.04175	0.06082	0.50173	0.42397	0.06481
	5	0.20374	0.11308	0.11125	**0.11110**	0.11128	0.22189	0.73685	6.52117	19.87954
	8	46.46431	0.17984	**0.17513**	0.17541	0.17601	0.23603	0.72480	0.52039	75.18619
DTLZ2	3	**0.05387**	0.05799	0.05804	0.05801	0.05800	0.07318	0.54279	0.54641	0.07182
	5	0.20471	0.19403	**0.19363**	0.19368	0.19368	0.32648	0.69062	0.93890	0.31393
	8	0.55342	0.40062	0.39802	0.39575	**0.39572**	0.46026	0.94291	0.99204	1.90946
DTLZ3	3	**0.05444**	0.06261	0.05908	0.05824	0.05848	0.07349	0.54419	0.54800	0.07194
	5	2.00650	0.19601	0.19496	**0.19384**	0.19400	0.32551	0.70566	40.98681	116.19480
	8	189.45920	0.41225	0.40224	**0.39694**	0.46660	0.47438	0.94647	1.23378	348.09573
DTLZ4	3	**0.05411**	0.07550	0.10791	0.05800	0.45495	0.47158	0.83789	0.71489	0.07012
	5	0.19999	0.19378	0.19373	**0.19372**	0.33507	0.45264	0.82880	0.89434	0.22875
	8	0.59800	0.39672	0.39597	**0.39534**	0.53322	0.64479	0.95178	1.00074	2.11783
WFG1	3	0.37755	0.21258	0.18074	0.18377	0.20233	**0.07600**	0.20087	0.15597	0.16604
	5	0.50043	0.29117	0.20606	0.17134	0.19663	**0.08683**	0.18288	0.18297	0.26815
	8	0.56936	0.16839	0.07692	**0.06678**	0.08509	0.08045	0.10808	0.12427	0.33417
WFG2	3	0.28908	0.04072	**0.03577**	0.04866	0.08872	0.08739	0.17910	0.12579	0.05805
	5	0.06711	0.05691	**0.05685**	0.08325	0.10423	0.15136	0.21243	0.20765	0.12767
	8	0.25341	**0.07015**	0.08495	0.09183	0.09860	0.11937	0.13764	0.13030	0.19386
WFG3	3	0.19028	0.15399	0.28832	0.05425	**0.03745**	0.04070	0.20844	0.19232	0.05006
	5	0.28116	0.09697	0.12176	0.12018	**0.08618**	0.15235	0.34998	0.28723	0.10195
	8	0.36845	0.23351	0.56029	**0.14305**	0.22451	0.33536	0.56095	0.43524	0.15998
WFG4	3	0.06435	**0.05818**	0.05823	0.07217	0.07700	0.09484	0.52334	0.25250	0.07274
	5	**0.17257**	0.19213	0.19223	0.26733	0.30864	0.41147	0.63375	0.42761	0.18244
	8	**0.37202**	0.39954	0.39905	0.51790	0.72445	0.51843	0.85709	0.59237	0.37909
WFG5	3	0.06355	0.06216	**0.06212**	0.07543	0.07569	0.10004	0.52875	0.24320	0.07718
	5	0.18643	0.18937	0.18935	0.25529	0.29036	0.40381	0.65914	0.41589	**0.18139**
	8	**0.36439**	0.39141	0.39123	0.51273	0.67067	0.51038	0.81440	0.48871	0.36793
WFG6	3	0.07017	0.06237	**0.06236**	0.07542	0.08158	0.09964	0.53091	0.24512	0.08111
	5	0.24645	**0.18939**	0.18942	0.26168	0.32816	0.40693	0.67423	0.41625	0.19635
	8	**0.38894**	0.39279	0.39211	0.52623	0.84861	0.52593	0.92164	0.70887	0.40164
WFG7	3	0.06707	0.05858	**0.05843**	0.07272	0.10435	0.09461	0.53919	0.25365	0.07482
	5	0.21572	**0.19302**	0.19308	0.26131	0.34346	0.40967	0.67685	0.42667	0.22350
	8	**0.37702**	0.39970	0.39841	0.50986	0.81487	0.52613	0.92975	0.61293	0.43800
WFG8	3	0.08231	0.06858	**0.06826**	0.07974	0.08798	0.10758	0.53692	0.50862	0.09200
	5	0.19660	0.19572	**0.19568**	0.27004	0.31288	0.51613	0.70712	0.51826	0.21824
	8	**0.39016**	0.41691	0.41495	0.49936	0.80811	0.54876	0.92428	0.79070	0.43170
WFG9	3	0.07303	0.06403	**0.06323**	0.07385	0.10025	0.09920	0.50142	0.26204	0.08311
	5	0.19227	**0.18615**	0.18634	0.24683	0.29613	0.47733	0.66154	0.44104	0.21086
	8	0.41182	0.39688	**0.39539**	0.51814	0.71655	0.53759	0.85700	0.67375	0.45885