Optimization: An Introduction

Tapabrata Ray

MDO Group, UNSW, Canberra.

http://seit.unsw.adfa.edu.au/research/sites/mdo/index.htm

Acknowledgements: All former and current postgraduate students, collaborators and funding agencies.

Overview of Optimization

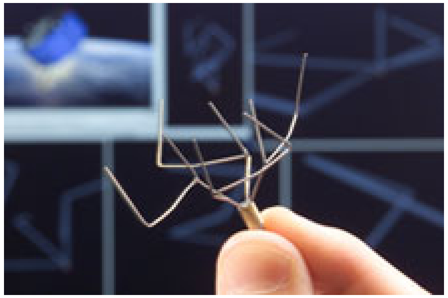
Constrained Optimization

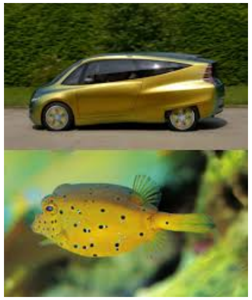
Many Objective Optimization

Shape Optimization Examples

Application Snapshots

Basic Premise





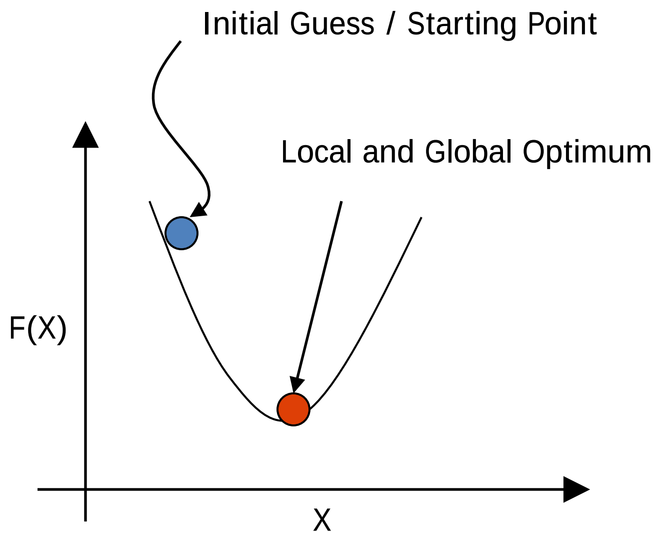
The unusual looking NASA ST5 antenna design was lighter, stronger and consumed less power. (Source: http://www.nasa.gov/centers/ames/research/exploringtheuniverse/borg.html).

The latest Boeing 787 Dreamliner is claimed to be quieter, more efficient and more environment friendly when compared with existing designs. (Source: http://www.compositestoday.com/2012/11/boeing-steps-up-787-dreamliner-production/)

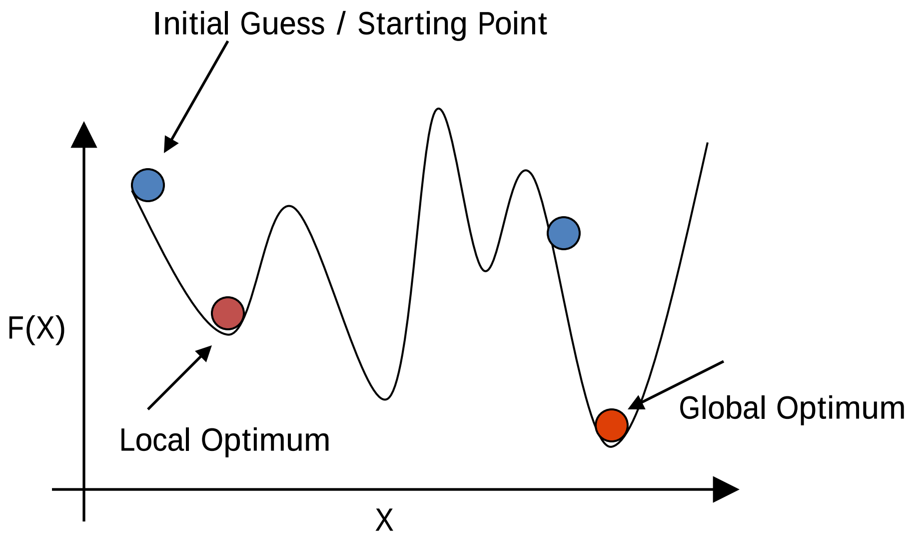
The Mercedes Benz Car design was inspired by drag characteristics of box-fish. (Source: http://webecoist.momtastic.com/2011/03/21/marine-muse-12-more-sea-inspired-designs-inventions/)

- Refers to Minimization or Maximization of one or more objectives.
- Variables: Continuous, Discrete, Integers.
- Constraints: Linear, Nonlinear: Inequalities and Equalities.
- Usually in the context of engineering, problems tend to have highly nonlinear objectives and constraints. Such objectives and constraints are often evaluated using computationally expensive simulations.

Basic Premise



Unimodal Function



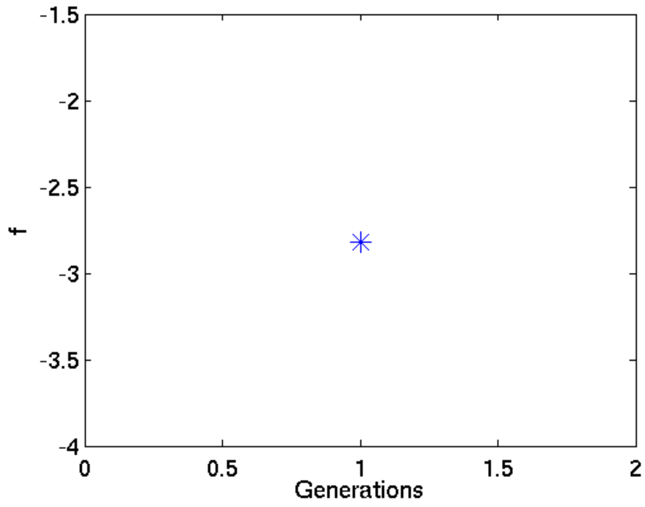
Multimodal Function

No Algorithm Can Guarantee to Locate Global Optimum for Multimodal Functions.

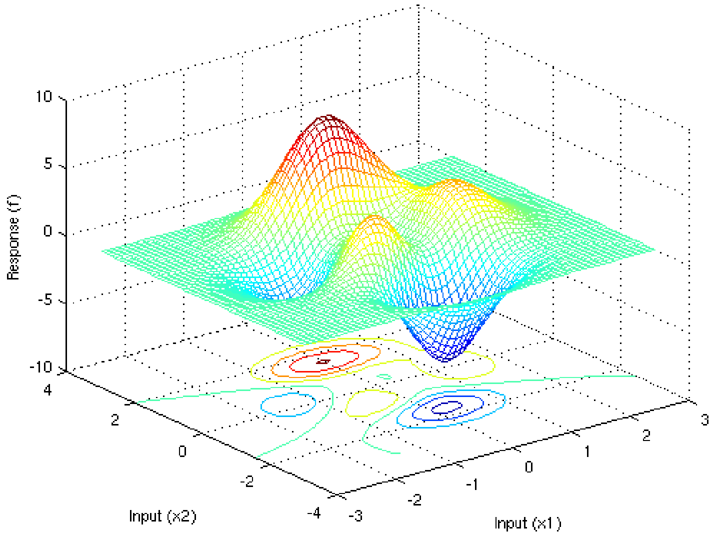
Gradient Based Algorithms Can Guarantee to Locate Local Optimum.

Zero Order Methods Can only Locate a *Good Solution* which may not even be a Local Optimum.

1. Generate a set of M solutions.
2. Identify better solutions as parents.
3. Combine the parents to create M child solutions.
4. Combine the original set of solutions (M) and child solutions (M).
5. Select M solutions from the above set of 2M solutions.
6. Repeat steps 2-4 till convergence condition is true.

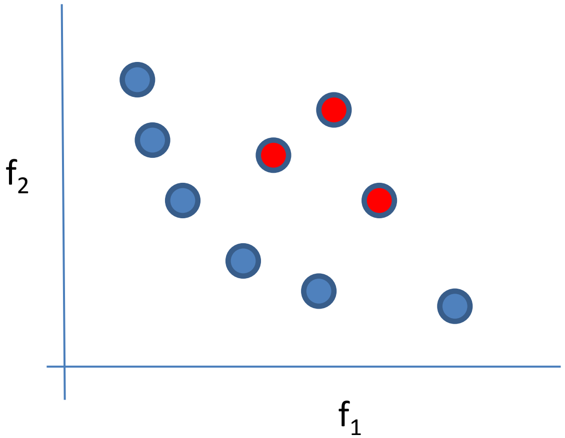


Progress Plot



Visualization

Multiobjective Optimization



Minimization and Maximization problems are interchangeable.

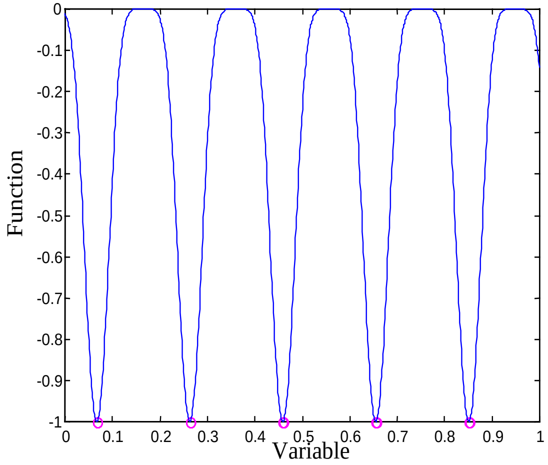
All discussions are in the context of minimization.

In Multio-bjective optimization, the interest is to find the non-dominated set of solutions that are close to the Pareto optimal set.

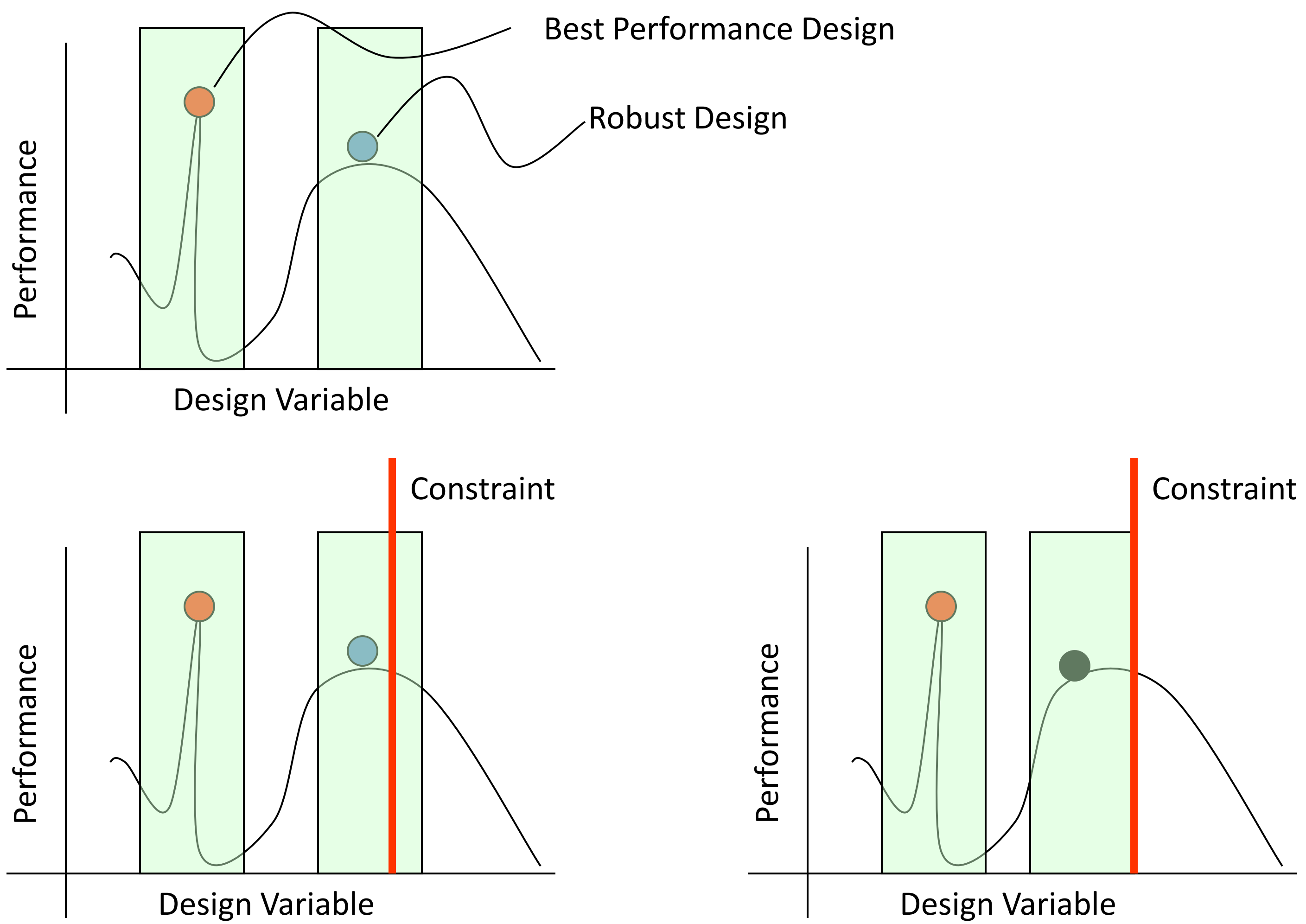
The ND set of solutions should have a good convergence and a good spread.

The solutions in blue are Non-dominated solutions. The solutions in red are dominated solutions.

Designers are interested in identifying multiple optima.



Basic Premise

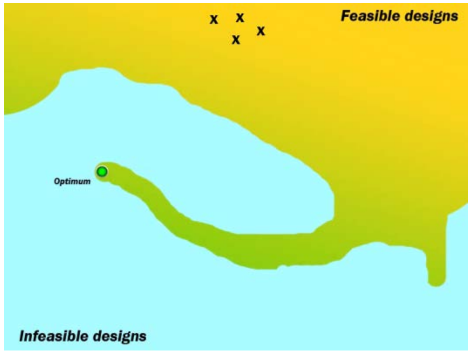


Notice that the Design Shifted away from the Constraint Boundary

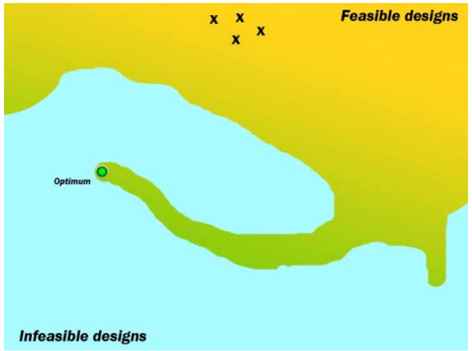
Constrained Optimization

Constraints and their Effects

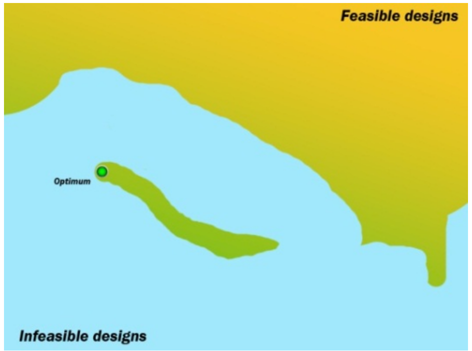
- Solutions to constrained optimization problems often lie on constraint boundaries.
- Most real coded population based stochastic algorithms intrinsically prefer a feasible solution over another.
- Proposed Infeasibility Driven Evolutionary Algorithm IDEA).



Real coded EA



Preserving Infeasible Solutions

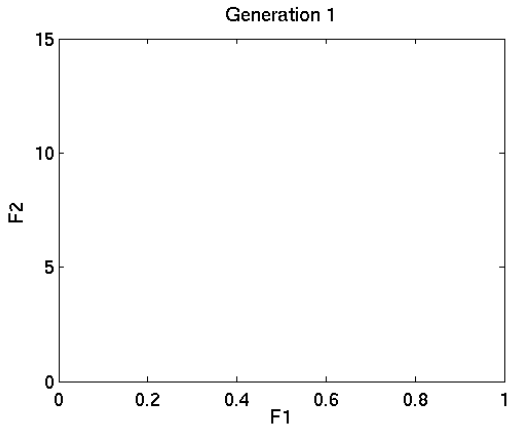


Disconnected Feasible Regions

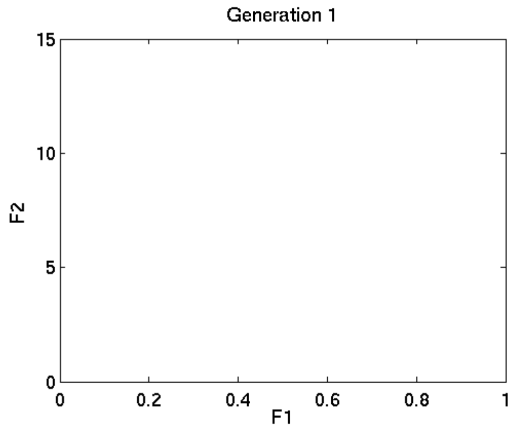
- Explicitly preserves a fraction of infeasible solutions across generations.
- Marginally infeasible solutions are preferred over feasible solutions.
- Offers a trade-off set of solutions with minimal constraint violation in addition to the set of feasible solutions.

Performance of IDEA

- Behaviour of IDEA on multi-objective optimization problems.



Real coded EA



IDEA

Ray, T., Singh, H.K., Isaacs, A., and Smith, W. Infeasibility Driven Evolutionary Algorithm for Constrained Optimization, *Constraint-Handling in Evolutionary Optimization*, *Studies in Computational Intelligence Series 198*, Eds, Efrén Mezura-Montes, Springer. pp 145-165., 2009.

Singh, H.K. , Ray, T. and Sarker, R. ,“Optimum oil production planning using infeasibility driven evolutionary algorithm,” Evolutionary Computation, In Press, (Accepted 09/201 1).

Many-Objective Optimization

An Illustrative Example

- In order to observe the process of evolution, we computed the average performance of the population i.e. average of the d_1 *and* d_2 *values for the individuals for DTLZ1 (3* objectives)
- One can observe from Fig. 8, that the average d_2 *converges to near zero* (i.e. near perfect alignment to the reference directions) while the average d_1 *measure* stabilizes at around 0.8 in the normalized plane indicating convergence to the Pareto front

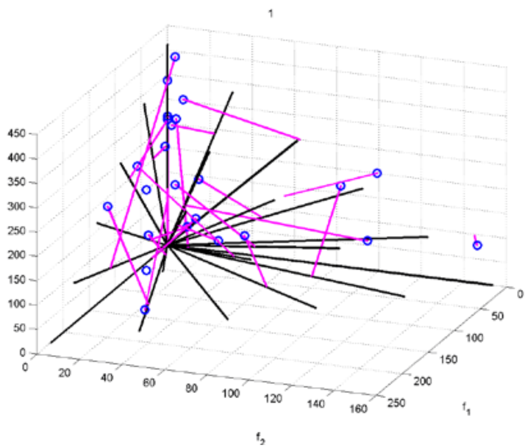


Fig:7. Evolving the best solutions with minimum d_1 and d_2 distances

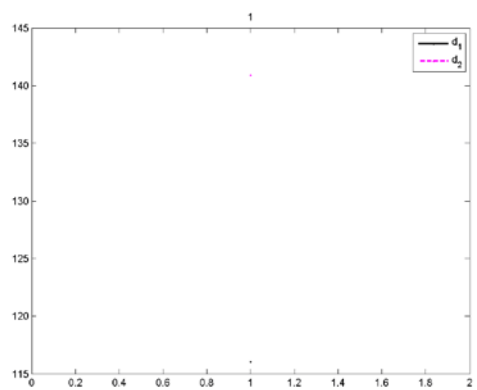


Fig:8. Converging the d_1 and d_2 measures over generations

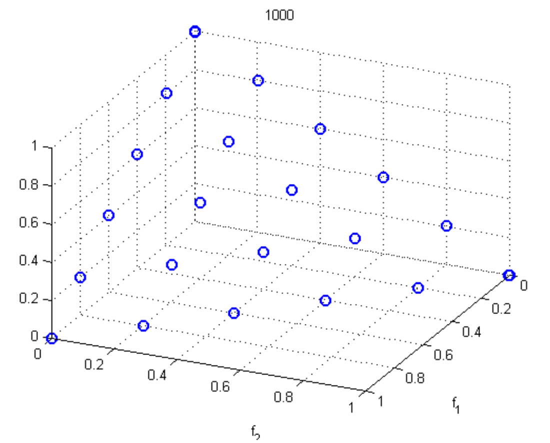


Fig:9. Final non-dominated solutions achieved for DTLZ1 problem

Shape Optimization

Flapping Wing Species

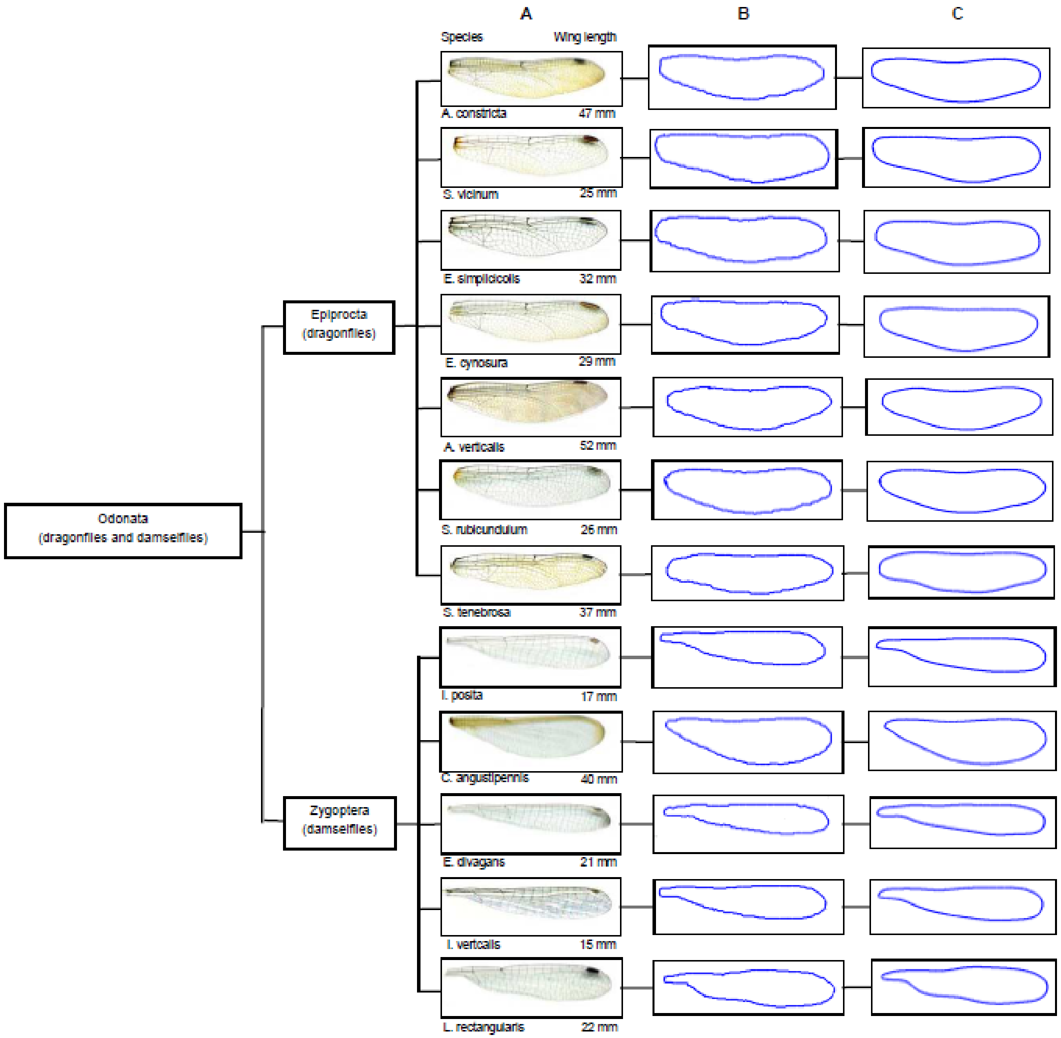
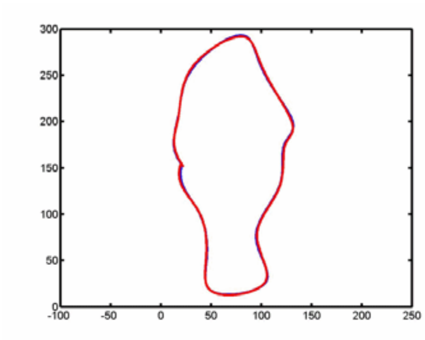


Fig.5 Dragonfly and Damselfly wing species and digitally extracted shapes' boundaries.

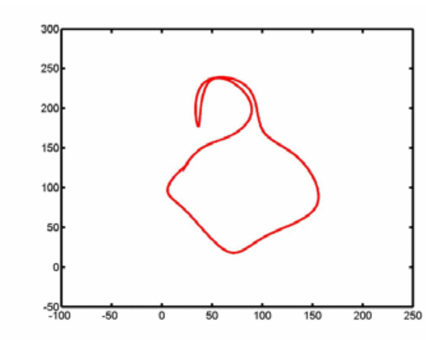
	Species	Length a, mm	Width b, mm	A.ratio a/b
Dragonfly	A. constricta	47	11.92	3.94:1
	S. vicinum	25	6.64	3.77:1
	E. simplicicolis	32	8.21	3.89:1
	E. cynosura	29	7.64	3.79:1
	A. verticalis	52	13.69	3.79:1
	S. rubicundulum	26	6.69	3.88:1
	S. tenebrosa	37	8.54	4.33:1
Damselfly	I. posita	17	3.36	5.06:1
	C. angustipennis	40	10.76	3.71:1
	E. divagans	21	3.59	5.84:1
	I. vertcalis	15	2.88	5.20:1
	L. rectangularis	22	4.68	4.70:1

Fig.6 Dragonfly and Damselfly wing dimensions with their aspect ratios

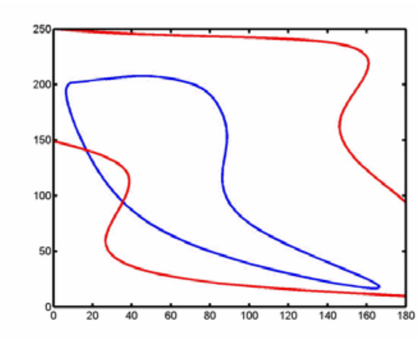
2D Examples



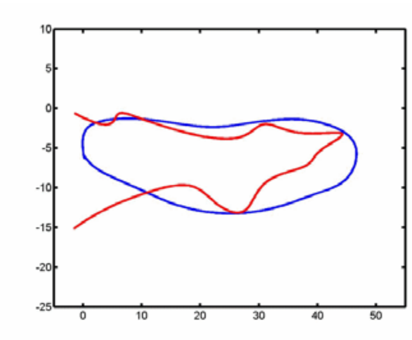
Simple fish



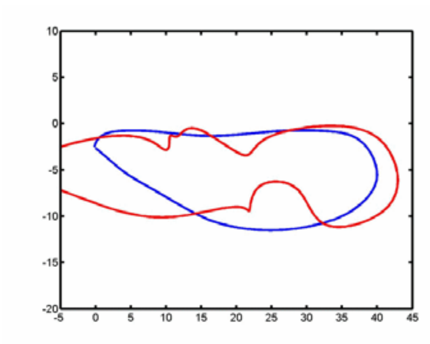
Stingray



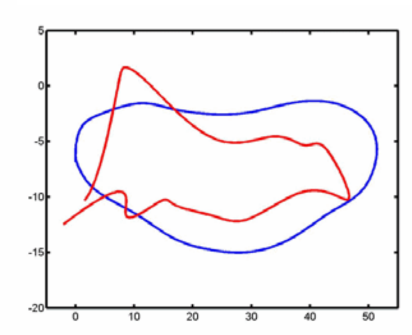
Airfoil



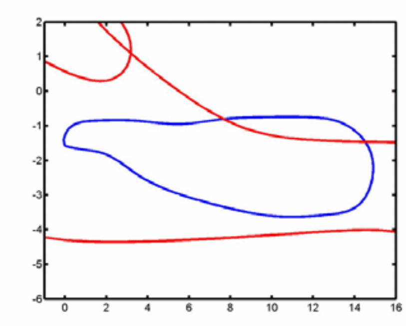
Dragonfly wing-1



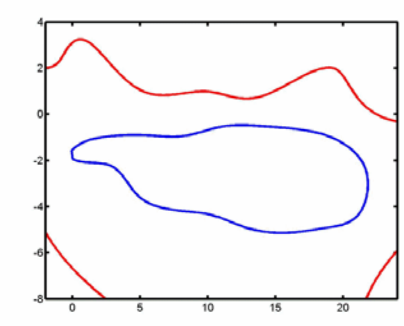
Dragonfly wing-2



Dragonfly wing-3



Damselfly wing-1



Damselfly wing-2

Fig.9 Evolution of generated shape (red) towards the target shape (blue)

Performance of Smart Repair Strategies 3D

- The optimum design of the 3D Flower vase (contains 676 points in x, y and z coordinates) after 100,000 function evaluations.

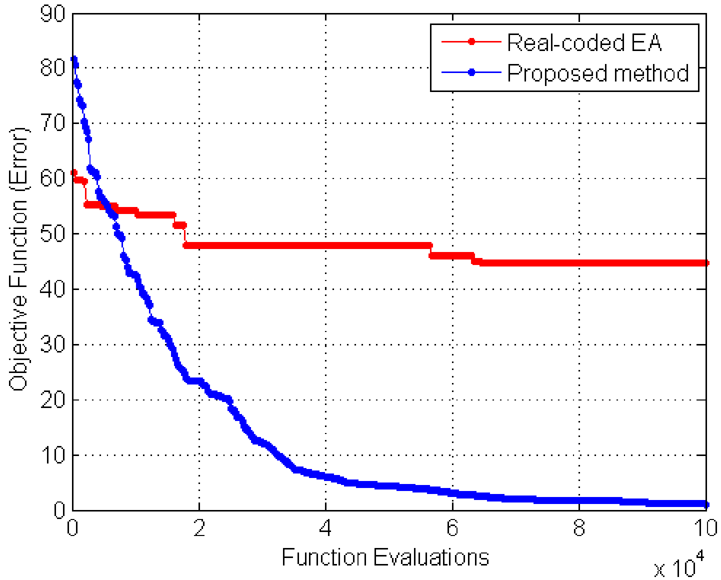
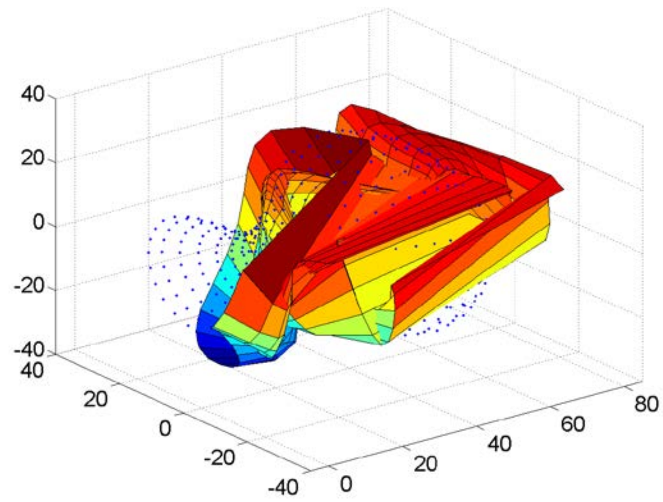


Fig.13 Progress plot of the best design for single objective matching error minimization



Ani.4 Evolution of 3D Flower vase (evolving surface) towards the target Flower vase (point cloud)

Method	Error Measurement	Best	Worst	Mean	Median	Std.
Real-coded EA	Max (Eucli,HD)	44.694	51.002	48.324	48.043	1.627
Proposed method	Max (Eucli,HD)	0.848	1.553	1.244	1.264	0.166

Tab. 4 Results for 3D Flower vase shape example

Real World Application (3D Ear Plug)

- A 3D ear plug has been considered to support real world product application.
- The evolution of shapes is very crucial component for optimization of shapes according to a patient's ear anatomy and canal.
- The target shape is extracted in the form of a point cloud via a 3D scan.
- The optimum design of the 3D ear plug (contains 841 points in x, y and z coordinates) after 150,000 function evaluations.

Fig. 14 Ear plugs

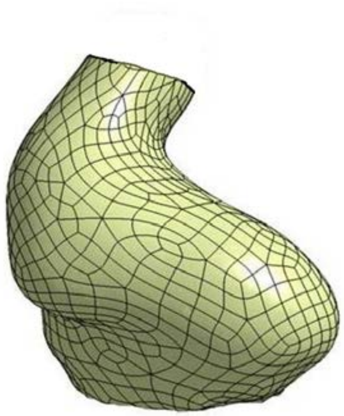


Fig. 15 3D scanned image

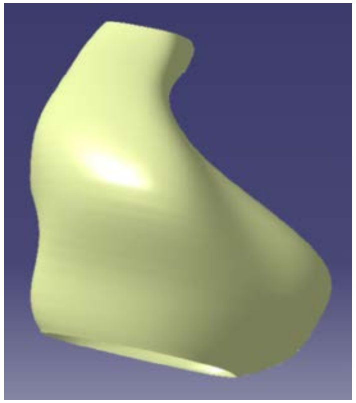
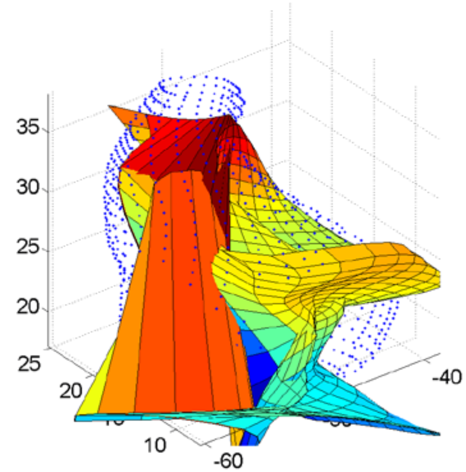


Fig.16 Target shape (CATIA model)



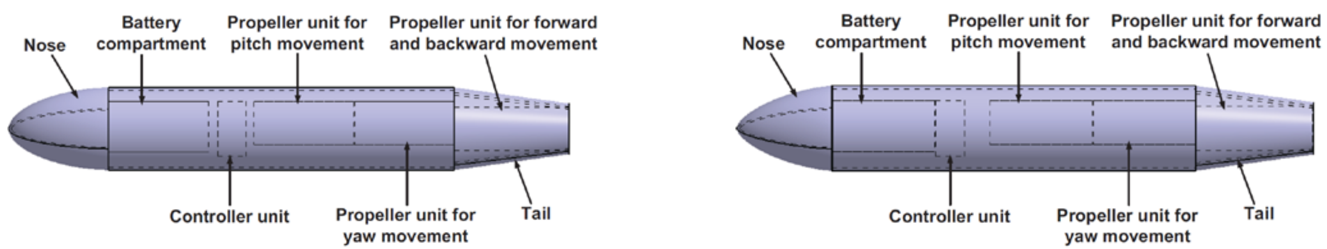
Ani.5 Ear plug evolutions

Approach 1: Results

- The best solutions of the two fidelity models are referred as the **Design-I** (best solution of the empirical-in-loop analysis) and **Design-II** (best solution of the CFD-in-loop analysis) toy submarines.

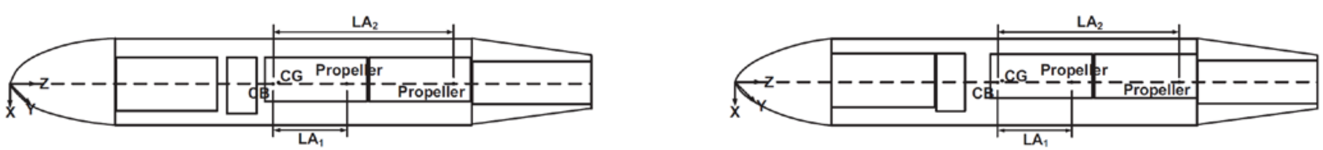
(a) Design-I (b) Design-II

Fig. CATIA models of the designed toy submarines



(a) Design-I (b) Design-II

Fig. Configurations of the designed toy submarines



(a) Design-I (b) Design-II

Fig. Longitudinal sections of the designed toy submarines

Six Inch Sub: Results

Fig. Bare hull of the best design

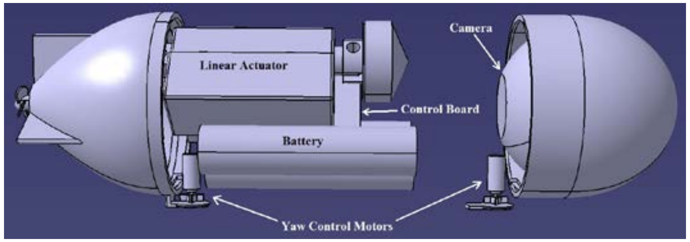
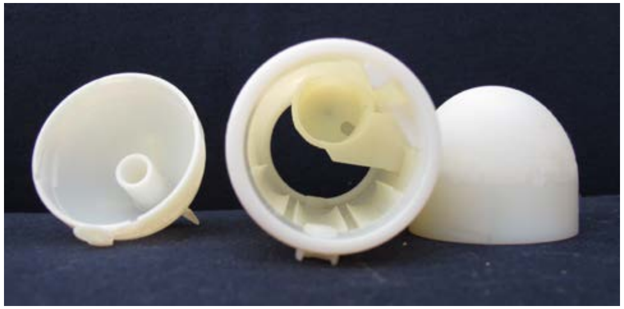


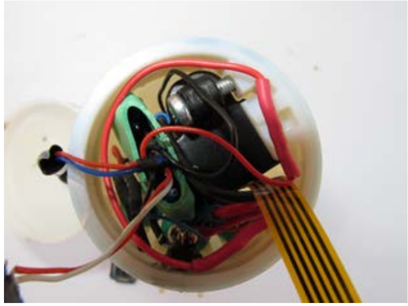
Fig. Internal image of the final design

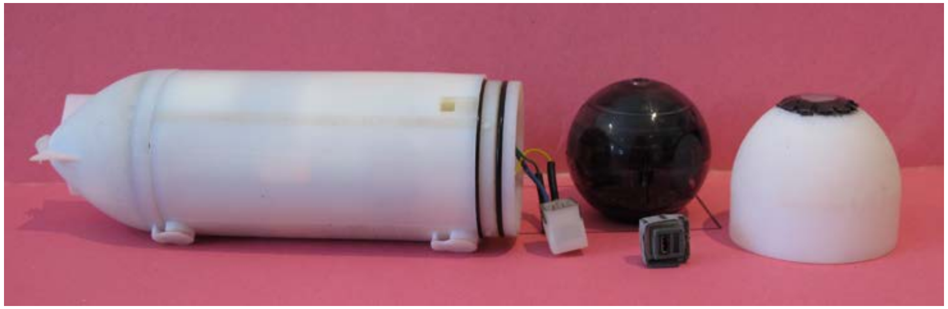
Table: Performance criteria of the optimized six inch sub

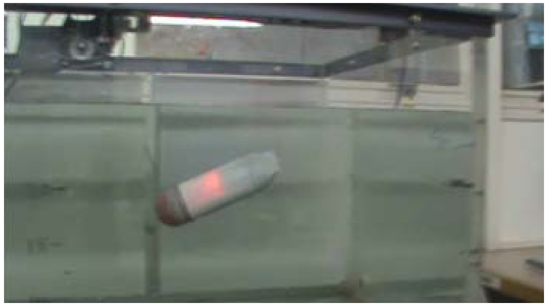
Vehicle Particulars			
Nose length	40 mm	Total mass of the vehicle	172.17 g
Mid-body length	78 mm	Max. inner square size	34.02 mm
Tail length	34.4 mm	First lever arm length	37.44 mm
Length overall	152.4 mm	Second lever arm length	38.41 mm
Outer diameter	51.1 mm	CG-CB separation	3.88 mm
L/D ratio	2.98	Nominal speed	0.2 m/s
Wetted surface area	0.021611 m^2	Drag (VT method)	0.006338 N
Displacement volume	0.000247 m^3	Drag (G&J method)	0.006733 N
Displaced water mass	0.247 kg	Drag (MIT method)	0.008178 N

Six Inch Sub: Fabrication





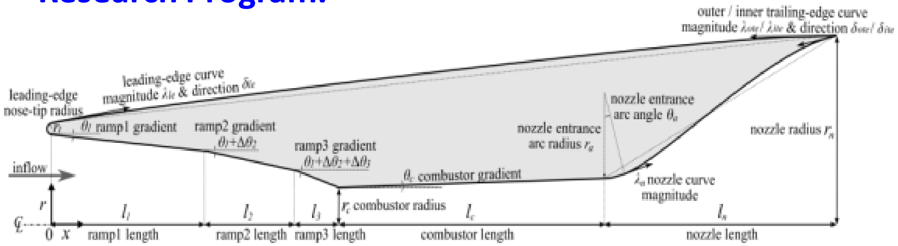


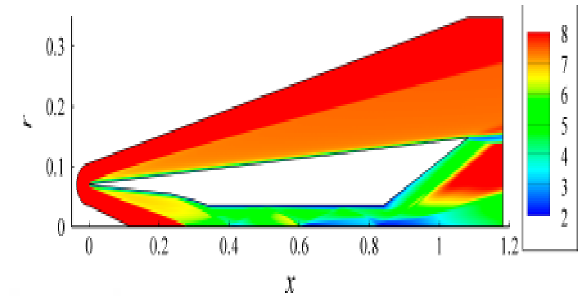


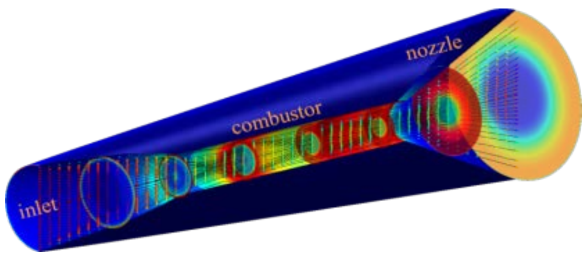
Design Optimization of Scramjet Geometries

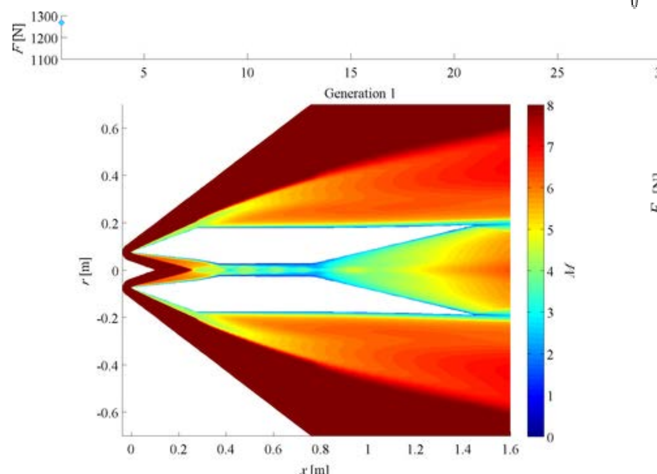
MDO RESEARCH GROUP
Multidisciplinary Design Optimization

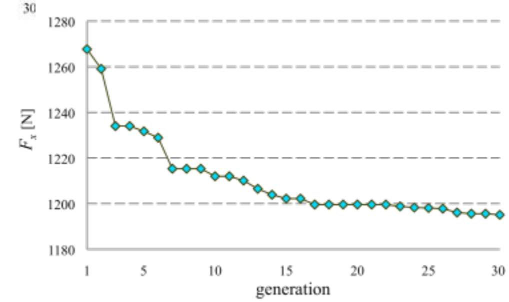
Full Flow-Path Design Optimization and Analysis of Axisymmetric Scramjets. Funded by Australian Space Research Program.

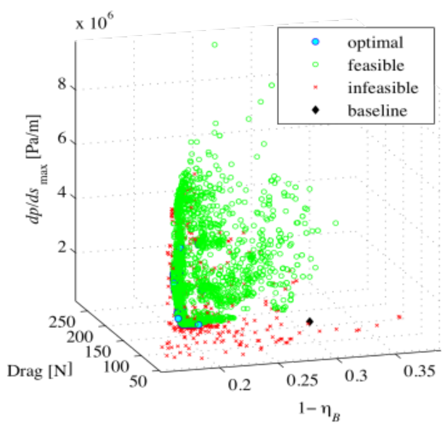


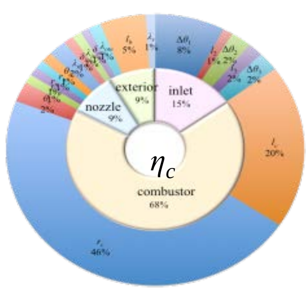












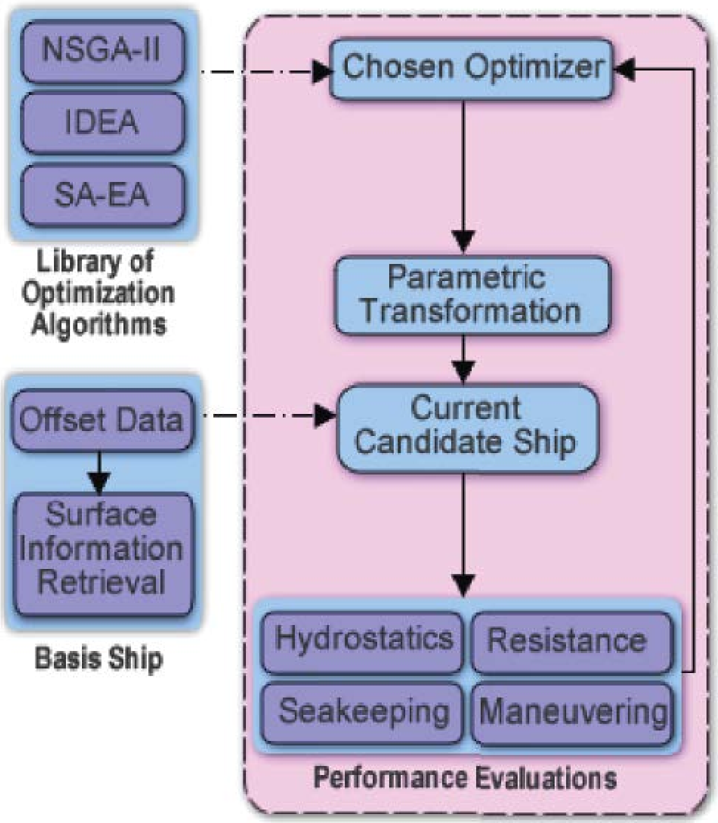
SCRAMSPACE

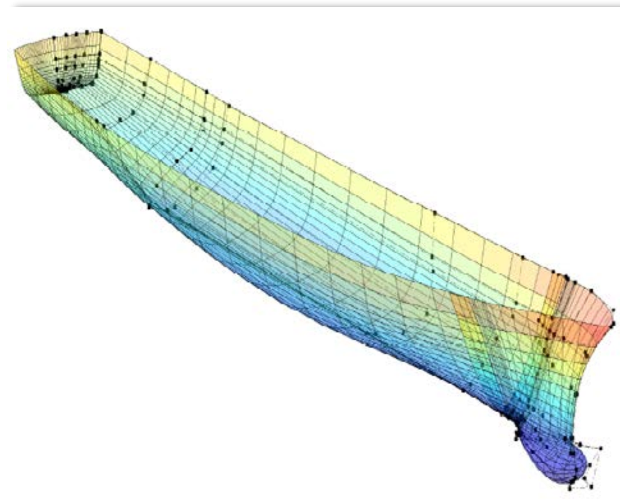
Ogawa,H. Brown,L. Boyce,R.R., and Ray,T., Multiobjective Design Optimization of Axisymmetric Scramjet Nozzle and External Components Considering Static Stability by using Surrogate Assisted Evolutionary Algorithms, International Society of Air-breathing Engines, ISABE 2011, September 12-16, 2011 Gothenburg, Sweden.

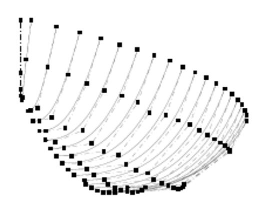
UNSW
THE UNIVERSITY OF NEW SOUTH WALES

Framework for Design Optimization of High Speed Craft

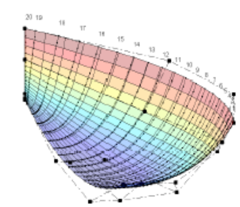
MDO RESEARCH GROUP
Multidisciplinary Design Optimization







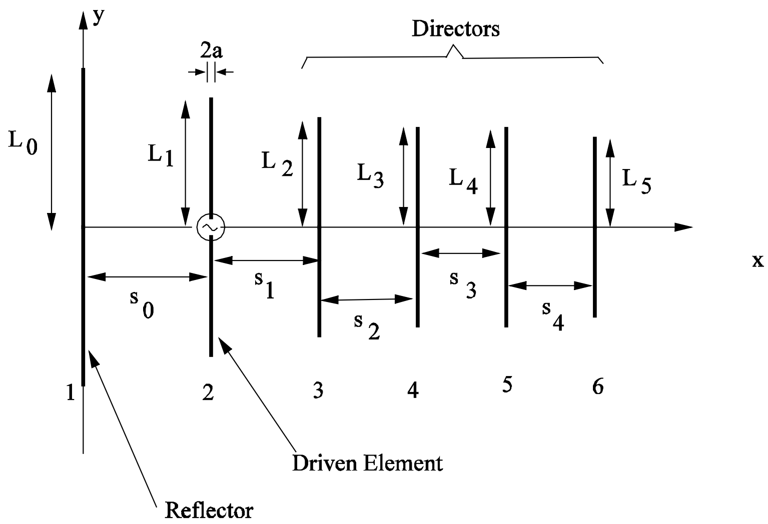
(b) Fitting station-directional data points ($\bar{\epsilon} = 0.34 \times 10^{-2}$)



(d) Surface defined from B-spline fitting process

1. A library of optimization algorithms
2. Surface information retrieval modules
3. Parametric transformation module
4. Hydrostatics and hydrodynamics module

Mohamad, A.F.A., Ray, T., and Smith, W.(2011), Uncovering Secrets Behind Low Resistance Planing Craft Hull Forms Through Optimization, *Engineering Optimization*.iFirst, 2011, pp. 1-13.



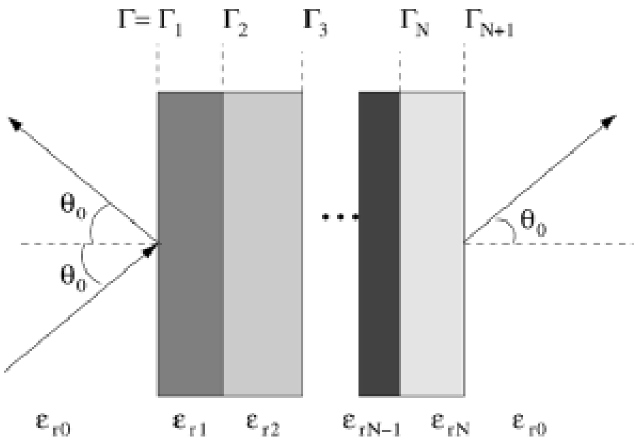
More than 1dBi improvement

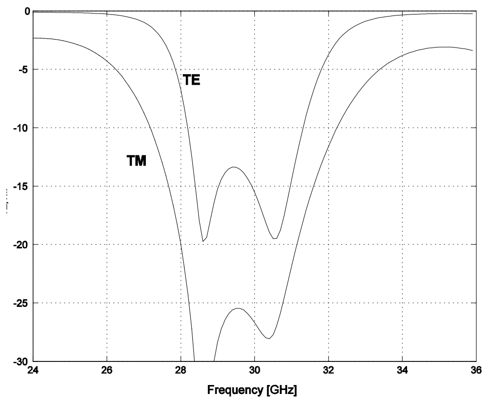
TABLE II
15-ELEMENT YAGI–UDA ANTENNA DESIGNS OBTAINED USING (a) GA FROM [7] AND (b) CI ALGORITHM

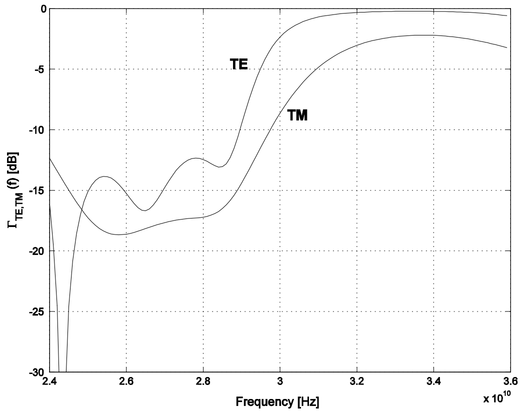
	GA optimized [7]		CI optimized	
Elements	Length, L_n	Spacing, S_n	Length, L_n	Spacing, S_n
1	0.236λ	-	0.235λ	-
2	0.230λ	0.249λ	0.227λ	0.196λ
3	0.221λ	0.155λ	0.224λ	0.238λ
4	0.205λ	0.185λ	0.215λ	0.142λ
5	0.216λ	0.191λ	0.204λ	0.231λ
6	0.210λ	0.252λ	0.212λ	0.447λ
7	0.210λ	0.442λ	0.206λ	0.395λ
8	0.189λ	0.431λ	0.203λ	0.371λ
9	0.191λ	0.362λ	0.201λ	0.441λ
10	0.200λ	0.205λ	0.202λ	0.433λ
11	0.204λ	0.268λ	0.206λ	0.445λ
12	0.215λ	0.414λ	0.196λ	0.365λ
13	0.174λ	0.197λ	0.189λ	0.359λ
14	0.199λ	0.130λ	0.203λ	0.429λ
15	0.204λ	0.362λ	0.196λ	0.390λ
Gain(dBi)	15.41		16.66	
Z (Ω)	50..64 – j 5.08		45.42 – j 5.74	

Venkatarayalu, N. and Ray, T. (2004). Optimum Design of Yagi-Uda Antennas Using Computational Intelligence, *IEEE Trans. On Antennas and Propagation*, Vol. 52, No. 7, pp. 1811- 1818, 2004.

Dielectric Filter Design







Bandpass Filter Design: Lower cutoff at 28 GHz and Upper cutoff at 32GHZ. Reflection coefficient is greater tha -5dB in stopband and less than -10dB in the passband. & layered dielectric.

Lowpass Filter Design: Cutoff frequency of 30GHZ.

Maximum of 15000 Design Evaluations.

Venkatarayalu, N., Ray, T. and Gan, Y.B., (2005). Multilayer Dielectric Filter Design Using a Multi-objective Evolutionary Algorithm, *IEEE Trans. On Antennas and Propagation*, Vol. 53, No. 11, pp. 3625-3632, 2005.